Two-channel Kondo model as a generalized one-dimensional inverse square long-range Haldane-Shastry spin model

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Majorana fermion representations of the algebra associated with spin, charge, and flavor currents have been used to transform the two-channel Kondo Hamiltonian. Using a path integral formulation, we derive a reduced effective action with long-range impurity spin-spin interactions at different imaginary times. In the semiclassical limit, it is equivalent to a one-dimensional Heisenberg spin chain with two-spin, three-spin, etc. long-range interactions, as a generalization of the inverse-square long-range Haldane-Shastry model. In this representation the elementary excitations are "semions", and the non-Fermi-liquid low-energy properties of the two-channel Kondo model are recovered.

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The two-channel Kondo model is known to have a non-Fermi liquid (non-FL) low-energy fixed point in the overscreening case, and has been put forward as a model to explain non-FL behavior observed in several quite different physical systems at low temperatures, such as certain heavy fermion alloys and two-level systems [1]. There are exact solutions for the ground state and thermodynamics of this model derived from the Bethe Ansatz [2,3], but there are still continuing efforts to find an intuitive understanding of the nature of excitations in the neighborhood of the low-energy fixed point. Numerical renormalization group [4], conformal field theory (CFT) [5], and bosonization approach [6–8] made a number of predictions for the many-body excitations at the fixed point, but they do not provide an intuitive interpretation of these excitations as it is possible at the FL fixed point of the single-channel Kondo model [9].

It was shown a long time ago that the single-channel Kondo model can be reduced to an inverse-square one-dimensional *Ising* model [10], which is a prototype classical model in statistical physics [11,12]. Moreover, such a reduced effective model had helped Anderson and coworkers to establish the correct FL behavior of the low-energy fixed point for the one-channel model in the early 70s [10,13], namely, the *Ising* spin-spin correlation function should behave as $\frac{1}{\tau^2}$ with τ as the imaginary time. On the other hand, the non-FL behavior of the two-channel Kondo model is characterized by the dynamic correlation function of the impurity spin, $< T_\tau \overrightarrow{S_d}(\tau) \cdot \overrightarrow{S_d}(0) > \sim \frac{1}{|\tau|}$, but its physical meaning, unfortunately, has not been fully understood yet. To our knowledge, the possible connection of the overscreened two-channel Kondo model with quantum spin models has not been explored so far.

In this Letter, the algebra of total spin, charge, and flavor currents of the two-channel Kondo model have been represented in terms of Majorana fermions, and in the path integral formulation, we derive a reduced equivalent quantum Heisenberg spin model, in which the impurity spins at different imaginary times are strongly correlated, including two-spin, three-spin, etc. long-range exchange interactions. In particular, the two-spin interaction has exact the same form as the integrable inverse-square one-dimensional Heisenberg spin chain — the so-called Haldane-Shastry (HS) model [14], while the three-spin and four-spin interaction parts, etc. are natural generalizations of the HS model. The non-FL fixed point action of the two-channel Kondo model is identified with the two-spin interaction part (the HS model), and the elementary excitations of this low-energy non-FL fixed point are spinons ($S = 1/2$ objects) obeying semion (half-fractional) statistics intermediate between bosons and fermions [15]. Actually, the two-spin long-range interaction part forms *an ideal semion gas*, while the three-spin long-range term is a *dangerous irrelevant interaction*, leading to important corrections to the thermodynamic properties. The fourth and higher order terms are irrelevant variables. The non-FL properties of the two-channel Kondo model are thus recovered.

We start with the Hamiltonian of the two-channel Kondo model in the form,

$$\begin{aligned} H &= H_0 + H_I \\ H_0 &= \frac{v_f}{2\pi} \sum_{j=1}^{2} \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{+\infty} dx\ \psi^\dagger_{j,\sigma}(x)(i\partial_x)\psi_{j,\sigma}(x) \\ H_I &= \sum_{a=x,y,z} J_a S^a_d J^a_s(0), \end{aligned} \tag{1}$$

where we have retained only the s-wave scattering, linearized the fermion spectrum, and replaced the incoming and outgoing waves with two left-moving electron fields $\psi_{j,\sigma}(x)$. $J^a_s(x)$ are the conduction electron spin current operators: $J^a_s(x) = \sum_{j,\sigma,\sigma'} : \psi^\dagger_{j,\sigma}(x) s^a_{\sigma,\sigma'} \psi_{j,\sigma'}(x) :$, where s^a being spin-1/2 matrices, :: means normal ordering. We introduce charge and flavor currents: $J_c(x) = \sum_{j,\sigma} : \psi^\dagger_{j,\sigma}(x)\psi_{j,\sigma}(x) :$ and $J^a_f(x) = \sum_{j,j',\sigma} : \psi^\dagger_{j,\sigma}(x) t^a_{j,j'} \psi_{j',\sigma}(x) :$, where $t^a_{j,j'}$ are generators of an SU(2) symmetry group. Following Affleck and Ludwig [5], the free part of the Hamiltonian can be rewritten

as a sum of three commuting terms by the usual point-splitting procedure (Sugawara construction):

$$H_0 = \frac{v_f}{2\pi}\int_{-\infty}^{+\infty} dx\left[\frac{1}{8} : J_c(x)J_c(x) : + \frac{1}{4} : \vec{J}_f(x)\cdot\vec{J}_f(x) : + \frac{1}{4} : \vec{J}_s(x)\cdot\vec{J}_s(x) :\right], \quad (2)$$

while the interaction term is expressed only in terms of the electron spin currents and the impurity spin. The information about the number of channels is contained in the commutation relations obeyed by the spin currents

$$[J_s^a(x), J_s^b(x')] = i\epsilon^{abc}J_s^a(x)\delta(x-x') + \frac{ki}{4\pi}\delta_{a,b}\delta'(x-x')$$

indicating that $J_s^a(x)$ form an SU(2) level $k = 2$ Kac-Moody algebra. Meanwhile, the charge and flavor currents satisfy

$$[J_c(x), J_c(x')] = 2ki\delta'(x-x'),$$
$$[J_f^a(x), J_f^b(x')] = i\epsilon^{abc}J_f^a(x)\delta(x-x') + \frac{ki}{4\pi}\delta_{a,b}\delta'(x-x').$$

They form a U(1) Kac-Moody and another SU($k = 2$) level-2 Kac-Moody algebra, respectively.

It is now quite natural to introduce a Majorana representation of the spin current operators,

$$\begin{aligned} J_s^x(x) &= -i\chi_2(x)\chi_3(x), \\ J_s^y(x) &= -i\chi_3(x)\chi_1(x), \\ J_s^z(x) &= -i\chi_1(x)\chi_2(x), \end{aligned} \quad (3)$$

where $\chi_1(x), \chi_2(x)$, and $\chi_3(x)$ are left-moving free Majorana fermion fields, and it can be shown to reproduce the SU(2) level-2 Kac-Moody commutation relations. It is important to note that this representation is only appropriate for the two-channel model as it leads to a level-2 algebra. It would be *inappropriate* for the single-channel Kondo model where the corresponding spin current generates a level-1 algebra.

In a similar way, we can also introduce Majorana representations for the flavor currents

$$\begin{aligned} J_f^x(x) &= -i\chi_2'(x)\chi_3'(x), \\ J_f^y(x) &= -i\chi_3'(x)\chi_1'(x), \\ J_f^z(x) &= -i\chi_1'(x)\chi_2'(x), \end{aligned} \quad (4)$$

which reproduces the commutation relations satisfied by the flavor currents, and $J_c(x) = -2i\chi_4'(x)\chi_5'(x)$ representing the charge current operator. Note that χ_α' with $\alpha = 1,2,3,4,5$ are also left-moving free Majorana fermion fields. It is well-known that the dynamics of charge, flavor, and spin are completely determined by the commutation relations of these current operators. Though the spin currents of the two-channel Kondo model can be represented in terms of three Majorana fermion fields $\chi_\alpha(x)$ ($\alpha = 1,2,3$), they can not be given any simple physical interpretation in terms of the original conduction electrons $\psi_{j,\sigma}(x)$.

Now using these current operators, the Hamiltonian (2) is presented as a quartic form in the Majorana fields. This form is convenient if one pursues a purely algebraic approach as in the CFT [5]. However, for our purpose it is more convenient to perform an *inverse* Sugawara construction using again the point-splitting procedure, and rewrite the terms quartic in the Majorana fermions as kinetic energy terms which are quadratic [16,17]:

$$\begin{aligned} : J_c(x)J_c(x) :&= 4\sum_{\alpha=4}^{5}\chi_\alpha'(i\partial_x)\chi_\alpha'(x); \\ : \vec{J}_f(x)\cdot\vec{J}_f(x) :&= 2\sum_{\alpha=1}^{3}\chi_\alpha'(i\partial_x)\chi_\alpha'(x); \\ : \vec{J}_s(x)\cdot\vec{J}_s(x) :&= 2\sum_{\alpha=1}^{3}\chi_\alpha(i\partial_x)\chi_\alpha(x). \end{aligned} \quad (5)$$

The model Hamiltonian is thus transformed into the following two parts [18],

$$\begin{aligned} H_c + H_f &= \frac{v_f}{4\pi}\sum_{\alpha=1}^{5}\int_{-\infty}^{+\infty} dx\ \chi_\alpha'(x)(i\partial_x)\chi_\alpha'(x), \\ H_s &= \frac{v_f}{4\pi}\sum_{\alpha=1}^{3}\int_{-\infty}^{+\infty} dx\ \chi_\alpha(x)(i\partial_x)\chi_\alpha(x) \\ &\quad - \frac{iJ}{2}\int_{-\infty}^{+\infty} dx\ \delta(x)\ \vec{S}_d\cdot\vec{\chi}(x)\times\vec{\chi}(x). \end{aligned} \quad (6)$$

$H_c + H_f$ describes the non-interacting charge and flavor degrees of freedom. It is invariant under the symmetry group $U(1)\otimes SU(2)_2$ described by the five free Majorana fermion fields $\chi_\alpha'(x)$ ($\alpha = 1,2,3,4,5$). H_s is the main part of the model and it describes the spin degrees of freedom with three left-moving Majorana fermion fields χ_α ($\alpha = 1,2,3$) interacting with the impurity spin. It has the $SU(2)_2$ symmetry so that the full Hamiltonian is described by eight different Majorana fermion fields.

In the two-channel model Hamiltonian, H_s will give rise to the essential low-energy physics of the model because it is the only part which includes the interaction. The reduced partition function of the model Hamiltonian with H_s can be written in the form of a functional integral:

$$Z = \oint D\widehat{\Omega}\int\prod_{\alpha=1}^{3}D\chi_\alpha\exp\left\{iS_d\omega\left(\widehat{\Omega}\right) - \int_0^\beta d\tau \left(\int_{-\infty}^{+\infty} dx\sum_{\alpha=1}^{3}\frac{1}{2}\chi_\alpha\partial_\tau\chi_\alpha + H_s\left[\widehat{\Omega},\chi_\alpha\right]\right)\right\}, \quad (7)$$

where the impurity spin part has been expressed in terms of a spin coherent state path integral [19], $\widehat{\Omega} = (\theta,\phi)$ is a unit vector describing the family of spin states $|S_d, m_d\rangle$,

the eigenstates of S_d^2 and S_d^z with eigenvalues $S_d(S_d+1)$ and m, respectively, and the periodic boundary condition is assumed for the spin vector variable.

$$iS_d\omega\left(\widehat{\Omega}\right) = iS_d\int_0^\beta d\tau(1-\cos\theta)\,\dot{\phi} \quad (8)$$

is known as the Berry phase of the spin history, which is a purely geometric factor and will play no essential role here. In the path-integral representation, the reduced Hamiltonian H_s is expressed as

$$H_s\left[\widehat{\Omega},\chi_\alpha\right] = \frac{v_f}{4\pi}\sum_{\alpha=1}^{3}\int_{-\infty}^{+\infty}dx\ \chi_\alpha(x,\tau)(i\partial_x)\chi_\alpha(x,\tau)$$
$$-\frac{iJS_d}{2}\int_{-\infty}^{+\infty}dx\ \delta(x)\ \widehat{\Omega}(\tau)\cdot\vec{\chi}(x,\tau)\times\vec{\chi}(x,\tau), \quad (9)$$

where $\chi_\alpha(x,\tau)$ $(\alpha=1,2,3)$ are real Grassmann variables corresponding to the three Majorana fermion fields and as far as χ_α are concerned, the path integrals over them are bilinear. The partition function can thus be rewritten in the following form

$$Z = \oint D\widehat{\Omega}\exp\left(iS_d\omega\left(\widehat{\Omega}\right)\right)\int\prod_{\alpha=1}^{3}D\chi_\alpha$$
$$\exp\left\{-\frac{1}{2}\int_0^\beta d\tau\int_{-\infty}^{+\infty}dx\ \Psi^\dagger(x,\tau)\widehat{M}\ \Psi(x,\tau)\right\}, \quad (10)$$

with a three-component vector $\Psi^\dagger(x,\tau) = (\chi_1(x,\tau), \chi_2(x,\tau), \chi_3(x,\tau))$ and its transposition $\Psi(x,\tau)$. The matrix is denoted by $\widehat{M} = (\partial_\tau - \overline{v}_f i\partial_x)I + iJS\delta(x)\widehat{M'}(\tau)$, where

$$\widehat{M'}(\tau) = \begin{bmatrix} 0, & -\Omega^z(\tau), & \Omega^y(\tau) \\ \Omega^z(\tau), & 0, & -\Omega^x(\tau) \\ -\Omega^y(\tau), & \Omega^x(\tau), & 0 \end{bmatrix}, \quad (11)$$

$\overline{v}_f = v_f/2\pi$, and I is a 3×3 unit matrix. Then we integrate out the variables χ_α and obtain an effective action which only contains the spin vector variables:

$$Z = Z_0\oint D\widehat{\Omega}\exp\left(iS_d\omega\left(\widehat{\Omega}\right) - S_{eff}\right),$$
$$S_{eff} = -\frac{1}{2}\mathrm{Tr}\ \ln\left[1 + iJS_d\delta(x)\widehat{G}\widehat{M'}(\tau)\right], \quad (12)$$

where $Z_0 = \frac{1}{2}\det\left[(\partial_\tau - \overline{v}_f i\partial_x)I\right]$ is the partition function of the non-interacting limit of H_s, and its free Majorana fermion propagator is given by $\widehat{G} = (\partial_\tau - \overline{v}_f i\partial_x)^{-1}$. Tracing here is taken over space, imaginary time, and the matrix indices. Using the identity

$$\mathrm{Tr}\ \ln(1+\widehat{A}) = -\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\mathrm{Tr}\ (\widehat{A})^n,$$

we obtain the general expression for the effective action

$$S_{eff} = \sum_{n=1}^{\infty}\frac{(-iJS_d)^n}{2n}\int_0^\beta d\tau_1 ...\int_0^\beta d\tau_n G(\tau_{12})G(\tau_{23})...$$
$$G(\tau_{n1})\mathrm{Tr}\left[\widehat{M'}(\tau_1)\widehat{M'}(\tau_2)......\widehat{M'}(\tau_n)\right], \quad (13)$$

where the space integration has been easily carried out due to the presence of the delta function so that the free Majorana fermion propagators are replaced by the local one, $G(\tau) = \frac{1}{\overline{v}_f}\frac{\pi/\beta}{\sin(\pi\tau/\beta)}$, and $\tau_{ij} = \tau_i - \tau_j$. Note that $\tau_1, \tau_2, ..., \tau_n$ are *unequal* imaginary times. In fact $S_{eff}^{(n)}$ is the contribution of a one-loop diagram made of n local propagators $G(\tau_{ij})$ and n local vertices $iJS\widehat{M'}(\tau_i)$. The factor $1/(2n)$ in front of each term is due to symmetry of the corresponding diagram. In the above derivation, the impurity spin is not specified to be 1/2 and only the spin of the conduction electrons has been assumed to 1/2. However, the following discussion will focus on the overscreening case ($S_d = 1/2$).

Up to second order of JS_d, the effective action is worked out as

$$S_{eff}^{(2)} = \frac{1}{2}(JS_d)^2\int_0^\beta d\tau_i\int_0^\beta d\tau_j G^2(\tau_{ij})\widehat{\Omega}(\tau_i)\cdot\widehat{\Omega}(\tau_j), \quad (14)$$

which describes a Heisenberg spin chain with an inverse-square long-range *antiferromagnetic* interaction between the impurity spins at two different imaginary times, and the sign of the original Kondo exchange coupling (ferromagnetic or antiferromagnetic) is not distinguishable here. Note that the spin coherent state path integral has assumed a periodic boundary condition for the spin variable so that the Heisenberg spins in fact sit on a circle of length β with exchange inversely proportional to the square of the distance between spins, which has the same form of the path-integral functional as the one-dimensional Heisenberg spin chain with an inverse-square long-range interaction, the HS model [14] in the *semiclassical* limit. Therefore, all the *static* properties of the HS model can be readily translated to the present model. (a) The low-energy states of $S_{eff}^{(2)}$ in the large-β limit are described by the chiral-SU(2) invariant $k=1$ Wess-Zumino-Witten model, which is a conformally invariant Gaussian field theory. $S_{eff}^{(2)}$ thus can represent a fixed point action of the two-channel Kondo model, and the elementary excitations are spinons, i.e., the $S=1/2$ particles instead of spin waves which are the elementary excitations of an ordered antiferromagnet. The spinons satisfy semion statistics intermediate between bosons and fermions, being an example of the *exclusion* statistics interpretation of fractional statistics [15].(b) $S_{eff}^{(2)}$ describes a free gas of spinons, being a fundamental model for gapless spin-1/2 antiferromagnetic spin model, and the dominant asymptotic spin-spin correlation function of $S_{eff}^{(2)}$ is algebraic with an universal exponent $\eta = 1$ without logarithmic corrections [14].

$$< T_\tau\widehat{\Omega}(\tau_i)\cdot\widehat{\Omega}(\tau_j) > \sim \frac{1}{\mid\tau_i - \tau_j\mid} \quad (15)$$

and the impurity spin variable $\widehat{\Omega}(\tau)$ thus acquires a *dynamic* scaling dimension 1/2. Such a behavior is also the universal spin-spin correlation function of the low-energy non-FL behavior of the two-channel Kondo model [5–8], leading to a marginal FL form [20] of the impurity spin spectrum: $\mathrm{Im}\chi_d(\omega + i0^+) \sim \tanh(\frac{\omega}{2T})$. (c). We can thus conclude that $S^{(2)}_{eff}$ represents the low-energy non-FL fixed point action of the spin part of the two-channel Kondo model.

The third order of JS_d can be viewed as a correction to the fixed point action, which has been derived as

$$S^{(3)}_{eff} = \frac{-i}{6}(JS_d)^3 \int_0^\beta d\tau_i \int_0^\beta d\tau_j \int_0^\beta d\tau_k \\ G(\tau_{ij})G(\tau_{jk})G(\tau_{ki})\widehat{\Omega}(\tau_i) \cdot \left(\widehat{\Omega}(\tau_j) \times \widehat{\Omega}(\tau_k)\right). \quad (16)$$

This term describes a Heisenberg spin chain with a long-range interaction of the impurity spins at three different imaginary times and is completely antisymmetric in its indices (ijk). In accordance with the low-energy non-FL fixed point action $S^{(2)}_{eff}$, this interaction term is irrelevant because it has a dynamic scaling dimension 3/2. However, it is this interaction that distinguishes the sign of the original Kondo exchange coupling, so that it is a *dangerous* irrelevant operator. It is conceivable that the non-FL thermodynamic properties of the two-channel Kondo model around the low-energy fixed point are derived from a perturbation theory of $S^{(3)}_{eff}$. For instance, the second order perturbation calculation of $S^{(3)}_{eff}$ gives rise to the extra low-temperature specific heat due to the presence of the impurity spin as $T \ln T$. The detailed calculations of the thermodynamic properties will be given in the future publication.

When the expansion is carried out to the fourth order, we obtain a four-spin long-range interaction of the impurity spins at four different times

$$S^{(4)}_{eff} = \frac{1}{4}(JS_d)^4 \int_0^\beta d\tau_i \int_0^\beta d\tau_j \int_0^\beta d\tau_k \int_0^\beta d\tau_l \quad (17) \\ G(\tau_{ij})G(\tau_{jk})G(\tau_{kl})G(\tau_{li})\widehat{\Omega}(\tau_i) \cdot \widehat{\Omega}(\tau_j)\ \widehat{\Omega}(\tau_k) \cdot \widehat{\Omega}(\tau_l),$$

which is clearly irrelevant as far as the low-energy fixed point is concerned, as its dynamic scaling dimension is 2. All higher-order terms thus contain no essential physics and can be neglected completely. Therefore, the reduced effective action will be given by $S_{eff} = S^{(2)}_{eff} + S^{(3)}_{eff}$, which is a natural generalization of the inverse-square long-range HS spin exchange model.

In summary, we use the Majorana fermion representation of the algebra of spin, charge, and flavor currents to transform the two-channel Kondo model. In a path integral formulation, we derive a reduced effective action of the two-channel Kondo model, which is a one-dimensional Heisenberg spin chains with two-spin, three-spin, etc. long-range interactions, as a natural generalization of the inverse-square long-range HS spin model. It is argued that the nontrivial two-channel Kondo physics in the low-energy regime can be reproduced from the first two terms of the spin action, and the other relevant issues are under investigation. As pointed out in Ref. [21], the infinite set of multiplicative degeneracy of HS model [15] is due to the hidden $SU(2)$ Yangian symmetry. Comparing our present formulation with the CFT treatment of the two-channel Kondo problem [5], this statement becomes apparent. After completing the present work, we become aware of a general review article [22] on exact results for highly correlated electron systems in one dimension, where some analogies of the inverse square long-range models to other interacting models are discussed within the framework of the Bethe-Ansatz.

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